Outage Probability Analysis of Multipath Fading Channels in Long Term Evolution-LTE

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ABSTRACT

3rd Generation Partnership Projects Long Term Evolution (3GPP-LTE) is a new standard for mobile broadband access that will meet the throughput and coverage requirements of a fourth generation cellular systems. The key goals for this evolution are increased peak data rate, improved spectrum efficiency, improved coverage and reduced latency. There are two typical ways to improve and measure a wireless communication system performance, which are bit error rate (BER and P(e)) and outage probability (Pout). In this paper, we focus on reducing Pout to improve our system performance. The outage performance method is based on channel capacity and mutual information.

Keywords: LTE; BER; Fading Channels; Outage Probability.

1. INTRODUCTION

Fourth generation (4G) Long Term Evolution-LTE wireless cellular communication systems must provide bandwidth-efficient, robust communication with low latency and supporting multiple users on broad band wireless channels. Since time varying channel conditions and thus time-varying channel capacity is an important features of 4G mobile communication systems and any future wireless communication systems, these systems should exhibit a high degree of adaptively on many levels in order to reach these goals. LTE is provides to improve throughput and spectral efficiency along with reducing the latency. The peak data rate requirement of LTE is 100Mbps for downlink and 50 Mbps for uplink. This can be achieved using a scalable bandwidth, multiple antenna elements and link adaptation. The bandwidth in LTE can vary from 1.4 MHz to 20MHz. LTE is also an Internet Protocol (IP)-packet based system with higher data rates and lower packet delay than previous generations [1-3].

Outage Probability is a key performance metric of wireless communication systems under co-channel interference (CCI). Outage occurs when the channel capacity is less than the transmission rate R, i.e. $\log \left(1 + \frac{SNR|h|^2}{\frac{R}{R}} \right) < R$ hence, the main idea of the performance improvement is increasing the channel capacity, which can be achieved by several transmission techniques such as LTE.

In this paper organized as follows, Section II explains about the statistical modeling of different fading channels, outage Probability analysis of fading channels explained in Section III and conclusion in section IV.

2. STATISTICAL MODELLING OF FADING CHANNELS

To understand wireless communications, it is necessary to explore what happens to the signal as it travels from the transmitter to the receiver. One of
the important aspects of this path between the transmitter and receiver is the occurrence of fading. The RF (radio frequency) signals with appropriate statistical properties can readily be simulated. Statistical testing can subsequently be used to establish the validity of the fading models frequently used in wireless systems.

Capacity of communication channels is the maximum data rate that can be transmitted over the channels such that arbitrary tolerable small error probabilities can be achieved. The channel capacity concept was initially defined by Shannon. Shannon defined the capacity in terms of mutual information as the maximum mutual information that can be achieved between the input and output of the channel.

Now let us consider the system capacity of the faded channel in the channel model is

\[ y[n] = h[n]x[n] + w[n] \]  
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Where \( h[n] \) represents the fading gain in the channel. Fading gain is modeled depending on different fading channels. \( x[n] \) represents the transmitted signal and \( w[n] \) a zero mean gaussian noise with variance and identically and independently distributed (iid) with respect to time. The capacity of the AWGN Channel is

\[ C_{awgn} = \log(1 + SNR) \text{ Bits/s/Hz} \]  
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In Rayleigh fading channels fading gain is assumed to be random but constant over time and delay constraint is small compared to the channel coherence time, the capacity of the channel can be written

\[ C = \log(1 + |h|^2 SNR) \text{ Bits/s/Hz} \]  
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A system is said to be outage if the transmitter sends data rate \( R \) is larger than the Channel capacity. Hence outage Probability is defined as follow

\[ P_{out} = \Pr[C < R] = \Pr \left[ \log(1 + |h|^2 SNR) < R \right] \]  
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A. Rayleigh Fading

The mobile antenna, instead of receiving the signal over one line-of-sight path, receives a number of reflected and scattered waves, as shown in Fig.1. Because of the varying path lengths, the phases are random, and consequently, the instantaneous received power becomes a random variable. In the case of an unmodulated carrier, the transmitted signal at frequency \( \omega \) reaches the receiver via a number of paths, the \( i^{th} \) path having an amplitude \( a_i \) and a phase \( \phi \). If we assume that there is no direct path or line-of sight (LOS) component, the received signal \( s(t) \) can be expressed as

\[ s(t) = \sum_{i=1}^{N} a_i \cos(\omega_i t + \phi_i) \]  
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The phase \( \phi \) depends on the varying path lengths, changing by \( 2\pi \) when the path length changes by a wavelength. Therefore, the phases are uniformly distributed over \([0, 2\pi]\). When there is relative motion between the transmitter and the receiver, eqn. (5) must be modified to include the effects of motion induced frequency and phase shifts. Let the \( i^{th} \) reflected wave with amplitude \( a_i \) and phase \( \phi \) arrive at the receiver from an angle \( \psi \) relative to the direction of motion of the antenna. The Doppler shift of this wave is given by

\[ \omega_{d_i} = \frac{\omega \nu}{c} \cos\psi \]  
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where \( \nu \) is the velocity of the mobile, \( c \) is the speed of light \( (3\times10^8 \text{ m/s}) \), and the \( \psi \)'s are uniformly distributed over \([0,2\pi]\). The received signal \( s(t) \) can now be written as

\[ s(t) = \sum_{i=1}^{N} a_i \cos(\omega_{d_i} t + \omega_i t + \phi_i) \]  
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Expressing the signal in phase and quadrature form, eqn. (3) can be written as

\[ s(t) = I(t) \cos \omega_i t - Q(t) \sin \omega_i t \]  
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where the in phase and quadrature components are respectively given as

\[ I(t) = \sum_{i=1}^{N} a_i \cos(\omega_{d_i} t + \phi_i) \]  
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\[ Q(t) = \sum_{i=1}^{N} a_i \sin(\omega_{d_i} t + \phi_i) \]  
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The envelope $R$ is given by

$$R = \sqrt{[I(t)]^2 + [Q(t)]^2}$$  \hspace{1cm} (11)

When $N$ is large, the in phase and quadrature components will be Gaussian [5]. The probability density function (pdf) of the received signal envelope, $f(r)$, can be shown to be Rayleigh given by

$$f(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad r \geq 0$$  \hspace{1cm} (12)

Varying the number of paths, it can be seen that the fading envelope in the absence of a line-of-sight path fits the Rayleigh distribution for as few as six paths. This was established by conducting chi-square tests for different values of $N$. Figure 2 shows the RF signals and envelopes for the case of a stationary mobile unit ($N=10$). The Rayleigh faded RF signal (Figure 2a) and envelope (Figure 2c) show that the signal strengths can fall below the average value (shown by the horizontal line in Figure 2c).

**B. Rician Fading**

The Rician distribution is observed when, in addition to the multipath components, there exists a direct path between the transmitter and the receiver. Such a direct path or line-of-sight component is shown in Fig.1. In the presence of such a path, the transmitted signal given in eqn. (3) can be written as

$$s(t) = \sum_{i=1}^{N-1} a_i \cos(\omega_i t + \phi_i) + k_d \cos(\omega_d t + \phi_d)$$  \hspace{1cm} (13)

Where the constant $k_d$ is the strength of the direct component, $\omega_d$ is the Doppler shift along the LOS path, and $\omega_i$ are the Doppler shifts along the indirect paths given by equation (2). The envelope in this case has a Rician density function given by [5]

$$f(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + k_d^2}{2\sigma^2}\right) J_0\left(\frac{r k_d}{\sigma}\right), \quad r \geq 0$$  \hspace{1cm} (14)

Where $J_0(\cdot)$ is the 0th order modified Bessel function of the first kind. The cumulative distribution of the Rician random variable is given as

$$F(r) = 1 - Q\left(\frac{k_d}{\sigma}, \frac{r}{\sigma}\right), \quad r \geq 0$$  \hspace{1cm} (15)

Where $Q(\cdot)$ is the Marcum’s Q function [4 & 6].

The Rician distribution is often described in terms of the Rician factor $K$, defined as the ratio between the deterministic power (from the direct path) and the diffuse signal power (from the indirect paths). $K$ is usually expressed in decibels as

$$K(dB) = 10 \log_{10} \left(\frac{k_d^2}{2\sigma^2}\right)$$  \hspace{1cm} (16)

In equation (16), if $k_d$ goes to zero (or if $k_d^2/2\sigma^2 \ll r^2/2\sigma^2$), the direct path is eliminated and the envelope distribution becomes Rayleigh, with $K(dB) = -\infty$.

To simulate the presence of a direct component, the received signal was modeled by eqn. (9). This meant that a term without any random phase needs to be added to the signal generated in the case of Rayleigh fading. The rest of the simulation was carried out as in the case of Rayleigh fading.

The RF signal and the envelope corresponding to $N = 10$ are shown in Figure 2b and Figure 2d. It is seen that the fluctuation in the envelope for Rician is much smaller than for the Rayleigh case (Figure 2c). The horizontal line in Figures (2c) and (2d) correspond to the mean value of the Rayleigh envelope.

The RF signals and demodulated envelopes for both Rayleigh and Rician cases for a mobile velocity of 25 m/s are compared in Figure 3. It is seen that the
signal envelope goes below the threshold (indicated by the horizontal line) in Figures (3c) and (3d) more often than in Figures (2c) and (2d). This increases the chances of loss of signal determined by the appearance of the envelope below the threshold when the mobile unit is in motion.

The envelope histogram and the Rayleigh fit to the envelope are shown in Figure 4. The Rayleigh density function was created by calculating the Rayleigh parameter from the moments of the envelope data corresponding to eqn. (9). The fit of the histogram of the data to Rician can be undertaken similarly.

![Fig. 2. RF signals and envelopes for stationary mobile](image)

(a) Rayleigh faded signal (b) Rician faded signal (c) Rayleigh envelope (d) Rician envelope

**C. Nakagami M-Distribution**

It is possible to describe both Rayleigh and Rician fading with the help of a single model using the Nakagami distribution [6]. The fading model for the received signal envelope, proposed by Nakagami, has the pdf given by

\[
f(r) = \frac{2m^m}{\Gamma(m)\Omega^m} r^{2m-1} \exp\left\{ -\frac{mr^2}{\Omega} \right\}, \quad r \geq 0 \tag{17}
\]

![Fig. 3. RF signals and envelopes for mobile moving at a velocity 25 m/s](image)

(a) Rayleigh faded signal (b) Rician faded signal (c) Rayleigh envelope (d) Rician envelope

Where \( \Gamma(m) \) is the Gamma function and \( m \) is the shape factor (with the constraint that \( m \geq \frac{1}{2} \)) given by

\[
m = \frac{E^2 \{ r^2 \}}{E \{ r^2 \} - \{ r^2 \}^2} \tag{18}
\]

The parameter \( \Omega \) controls the spread of the distribution and is given by

\[
\Omega = E \{ r^2 \} \tag{19}
\]

The corresponding cumulative distribution function can be expressed as

\[
F(r) = P \left( \frac{mr^2}{\Omega} \right) \tag{20}
\]

Where \( P(.) \) is the incomplete Gamma function. In the special case \( m = 1 \), Nakagami reduces to Rayleigh distribution. For \( m > 1 \), the fluctuations of the signal strength reduce compared to Rayleigh fading, and Nakagami tends to Rician.

No special simulation was necessary to test for the validity of Nakagami fading. Since Nakagami distribution encompasses both Rayleigh and Rician, the signal envelopes were tested against the Nakagami distribution using the chi-square test.

The Nakagami distribution seems to be a good fit for Rayleigh fading with an average value of the parameter \( m = 1 \) [6]. It also seemed to fit the Rician distribution between \( 1 < m < 2 \). Results for the Rayleigh and Nakagami fit are shown in Fig. 4.

**3. OUTAGE PROBABILITY**

In a fading radio channel, it is likely that a transmitted signal will suffer deep fades that can lead a complete loss of the signal or outage of the signal. The outage probability is a measure of the quality of the transmission in a mobile radio channel. Outage is said to occur when the received signal power goes below a certain threshold level [3, 4]. It
can be calculated as the integral of the received signal power \( p(t) \) as

\[
P_{out} = \int_{0}^{\infty} p(t) dt
\]  

(21)

Where \( P_{th} \) is the threshold power.

The procedure to find the outage probability is as follows:

1. Calculate the received signal power as given in equation 2.
2. 

\[
f(r) = \int_{0}^{\infty} \frac{r^2}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \frac{1}{\alpha \sqrt{2\pi}} \exp\left(-\frac{(\ln \sigma - \mu)^2}{2\lambda^2}\right) d\sigma
\]  

(22)

Where \( \sigma \) is the mode or the most probable value of the Rayleigh distribution, \( \lambda \) is the shape parameter of the lognormal distribution.

3. Set a threshold power level for the received signal relative to the average signal power.
4. Count the number of times in the sample interval that the received signal power goes below this threshold.
5. Using the basic concept of probability, the outage is then calculated by taking the ratio of the count in step 3 to the total number of samples.

For one received signal, we calculated the outage probabilities for various thresholds, and compared these values to those calculated analytically. The outage probabilities calculated analytically and through simulations were found to tally quite well. Fig. 5 shows the curves for the outage probability, calculated analytically and through simulations, for the Rayleigh fading case. As observed from Table I, the outage probability (averaged over 50 simulations) in a Rician channel is lower than that in a Rayleigh channel, which can be attributed to the presence of a line-of-sight path. Moreover, the probability of outage increases as the mobile velocity, or resulting Doppler shift, increases.

<table>
<thead>
<tr>
<th>Mobile velocity M/se</th>
<th>Outage Probability (Rayleigh)</th>
<th>Outage Probability (Rician)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.19149</td>
<td>0.09331</td>
</tr>
<tr>
<td>2</td>
<td>0.19193</td>
<td>0.09312</td>
</tr>
<tr>
<td>4</td>
<td>0.19246</td>
<td>0.09314</td>
</tr>
<tr>
<td>6</td>
<td>0.19303</td>
<td>0.09346</td>
</tr>
<tr>
<td>8</td>
<td>0.19339</td>
<td>0.09350</td>
</tr>
</tbody>
</table>

Table I. Comparison of outage probability for Rayleigh and Rician fading for a number of values of mobile velocities.

Fig. 4: The histogram of the simulated data and the correspondingly matched density functions.

Fig. 5: Outage probability for Rayleigh fading and stationary mobile. Simulated values are compared against theoretically computed outage values.
4. CONCLUSION

In this paper, we show the simple and straightforward concept of multipath fading channels and it easy to visualize the intricacies and understand the relationship between the different parameters involved in fading. Reducing outage probability (Pout) is to improve system performance. The outage performance method is based on channel capacity and mutual information.

REFERENCES


BIOGRAPHIES

Patteti Krishna: Completed B.Tech degree in Electronics and Communication Engineering and M.Tech degree with specialization of Digital Systems & Computer Electronics from JNT University, Hyderabad in 2005 and 2008. He is having 9 years of teaching experience and published papers in various International & National journals. At present Krishna is pursuing PhD in wireless Communications under the faculty of Electronics & Communication Engineering. His research interests are Signal processing for wireless communications, MIMO Communications and Multiuser MIMO Communications. He is a Member of IEEE, ISTE and IETE.

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