Multiuser Detection in Shadowed Fading Channels with Impulsive Noise

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ABSTRACT

In direct sequence code division multiple accesses systems (DS-CDMA), the signals are transmitted over multipath channels that introduce fading. Multipath fading along with multiple access interference and intersymbol interference degrades the system performance. Further, simultaneous presence of multipath fading and shadowing leads to worsening of wireless channels. Moreover, experimental results have confirmed the presence of impulsive noise in wireless mobile communication channels. Hence, this paper presents a technique for multiuser detection in DS-CDMA systems over shadowed fading channels in presence of impulsive noise. Approximate expression for average probability of error of an M-decorrelator is derived, for the demodulation of binary phase shift keying (BPSK) signals over shadowed fading channels, by modeling the channel with generalized K (GK) distribution. A new M-estimator is proposed for robustifying the detector and its performance is also studied and analyzed by evaluating probability of error with the derived expression. Simulation results reveal that the proposed multiuser detector performs better in fading and shadowing with heavy-tailed impulsive noise.

Keywords: Multiuser detection; fading channel; impulsive noise; GK distribution; probability of error.

I. INTRODUCTION

Recent research has explored the potential benefits of evolutionary optimization algorithms and their application to multiuser detection (MUD) for direct-sequence code division multiple access (DS-CDMA) systems [1]-[4]. An adaptive robust MUD technique for CDMA by implementing Huber’s M-estimator using genetic algorithm (GA) is presented in [3] and shown that it is robust against heavy-tailed impulsive noise. Recently, particle swarm optimization (PSO) algorithm has been applied for the MUD [4] to detect received data bit by optimizing an objective function.

The generalized-K (GK) fading channel has been received considerable attention as it can provide a good fit to different fading environments such as Nakagami-m and Rayleigh-Lognormal [5]. Experimental results have confirmed the presence of heavy-tailed impulsive noise in outdoor mobile communication channels, in radar and sonar systems and in indoor wireless communication channels [6]. Hence, this paper presents the implementation and performance analysis of proposed based M-estimator [7], [10], which performs well in the heavy-tailed impulsive noise, using PSO algorithm.

II. SYSTEM MODEL

Consider an L-user synchronous CDMA system, where each user transmits information by modulating a PN sequence over a single-path GK fading channel. The received signal over one symbol duration can be modeled as [8]

$$r(t) = \Re \left\{ \sum_{l=1}^{L} \sum_{i=0}^{M-1} h_l(i) a_l(t) e^{j \phi_l(i)} s_l(t - iT_s - \tau_l) \right\} + n(t)$$

where $\Re \{ \}$ denotes the real part, $M$ is the number of data symbols per user in the data frame of interest, $T_s$ is the symbol interval, $a_l(t)$ is the time-varying fading gain of the $l^{th}$ user’s channel, $\phi_l(i)$ is the time-
varying phase of the $l$th user’s channel, $b_l(i)$ is the $i$th bit of the $l$th user, $s_l(i)$ is the normalized signaling waveform of the $l$th user and $n(t)$ is assumed as a zero-mean complex two-term non-Gaussian noise [9]. For synchronous case (i.e., $r_1 = r_2 = ... = r_L = 0$), assuming that the fading process for each user varies at a slower rate that the magnitude and phase can taken to be constant over the duration of a bit, the received signal can be expressed in matrix notation as [9]

$$\mathbf{r}(i) = \mathbf{A}^{\theta}(i) + \mathbf{w}(i)$$

(2)

where $\mathbf{r}(i) = [r_1(i), ..., r_N(i)]^T$, $\mathbf{w}(i) = [w_1(i), ..., w_N(i)]^T$ and $\mathbf{\theta}(i) = [\theta_1(i), ..., \theta_L(i)]^T$. Here, $w_n(i)$ is a sequence of independent and identically distributed (i.i.d.) complex random variables whose in-phase and quadrature components are independent non-Gaussian random variables, $g_l(i)$ is the $l$th channel fading coefficient and

$$\mathbf{A}^{\theta} = [a_1^l, a_2^l, ..., a_L^l]$$

with $a_l^l = [a_{11}^l, a_{12}^l, ..., a_{1N}^l]$. It is assumed that the signal of each user arrives at the receiver through an independent, single-path fading channel. For the shadowed fading channels, $\alpha_l(i)$ are i.i.d. random variables with GK distribution given by

$$P_{\alpha_l}(\alpha_l) = \frac{2}{\Gamma(m)\Gamma(\mu)} \left( \frac{m\mu}{\Omega_0} \right)^{m+\mu} \frac{\alpha_l^{m+\mu}}{\Gamma(m+\mu)} K_{m-\mu} \left( 2 \sqrt{\frac{m\mu}{\Omega_0}} \alpha_l \right)$$

(3)

where, $m$ is the Nakagami fading parameter that determines the severity of the fading, $\mu$ represents the shadowing levels, $\Omega_0$ is the average SNR in a shadowed fading channel, $K_\mu(\cdot)$ is the modified Bessel function and $\Gamma(\cdot)$ is the Gamma function [9]. In M-estimates, unknown parameters $\theta_1, \theta_2, ..., \theta_L$ are solved by minimizing a sum of function, $\rho(\cdot)$ of the residuals [9]

$$\hat{\theta} = F(\theta(i)) = \arg\min_{\theta(i)} \sum_{l=1}^{L} \rho\left( r_n(i) - \sum_{l=1}^{L} [A]_{nl} \theta_l(i) \right) + \rho\left( r_n(i) - \sum_{l=1}^{L} [A]_{nl} \theta_l(i) \right)$$

(4)

where $\rho(\cdot)$ represents a specific penalty function that is symmetric positive-definite with a unique minimum at zero, and is chosen to be less increasing than square, $r_n(i)$ and $\theta_l(i)$ are the $n$th and $l$th elements of the vectors $\mathbf{r}(i)$ and $\mathbf{\theta}(i)$ respectively, $[A]_{nl}$ is the $n$th element of the matrix $A$, and $\mathfrak{R}(\cdot)$ denotes imaginary part. This paper considers the modified-Hampel based M-estimator to implement an M-decorrelating detector with penalty function [7]

$$\rho_{MH}(m) = \begin{cases} \frac{x^2}{2} \quad & \text{for } |x| \leq a \\
\frac{a^2 - |x|^2}{2} \quad & \text{for } a < |x| \leq b \\
\frac{-a^2 + b^2}{2} \exp\left(1 - \frac{b^2}{b^2 - |x|^2}\right) + d \quad & \text{for } |x| > b 
\end{cases}$$

(5)

where $a$, $b$ and $d$ are constants that depend on the robustness of the estimator [7]. It is assumed that the signal of each user arrives at the receiver through an independent, single-path fading channel. For the shadowed fading channels, $\alpha_l(i)$ are i.i.d. random variables with GK distribution given by

$$P_{\alpha_l}(\alpha_l) = \frac{2}{\Gamma(m)\Gamma(\mu)} \left( \frac{m\mu}{\Omega_0} \right)^{m+\mu} \frac{\alpha_l^{m+\mu}}{\Gamma(m+\mu)} K_{m-\mu} \left( 2 \sqrt{\frac{m\mu}{\Omega_0}} \alpha_l \right)$$

(6)

where, $m$ is the Nakagami fading parameter that determines the severity of the fading, $\mu$ represents the shadowing levels, $\Omega_0$ is the average SNR in a shadowed fading channel, $K_\mu(\cdot)$ is the modified Bessel function and $\Gamma(\cdot)$ is the Gamma function [9].
III. AVERAGE PROBABILITY OF ERROR FOR M-DECORRELATOR

In this section, the proposed M-estimator is presented and the average probability of error is derived for a single-path shadowed fading channel.

A) M-estimator

An M-estimator is a generalization of usual maximum likelihood estimates, used to estimate the unknown parameters \( \theta_1, \theta_2, \ldots, \theta_L \) (where \( \theta = Ab \)) by minimizing a sum of function \( \rho(\cdot) \) of the residuals [4]

\[
\hat{\theta} = \arg \min_{\theta \in \mathbb{R}^L} \sum_{j=1}^{N} \rho \left( r_j - \sum_{l=1}^{L} s_j^l \hat{\theta}^l \right)
\]

(7)

where \( \rho \) is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square and \( N \) is the processing gain. An influence function \( \psi(\cdot) = \rho^\prime(\cdot) \) is proposed (as shown in Fig. 1), such that it yields a solution that is not sensitive to outlying measurements, as [6]

\[
\psi_{PRO}(x) = \begin{cases} 
  x & \text{for } |x| < a \\
  \frac{a}{b} \exp \left( -\frac{x^2}{b^2} \right) & \text{for } |x| > b
\end{cases}
\]

(12)

where the choice of the constants \( a \) and \( b \) depends on the robustness measures.

B) Average probability of error

The asymptotic probability of error for the class of decorrelating detectors, for large processing gain \( N \), is given by [4]

\[
P_e \approx \Pr \left( \hat{\theta}_j < 0 | \theta_j > 0 \right) = \frac{Q \left( \frac{R_{ij}}{\sqrt{V L N^{-1} \mu}} \right)}{Q \left( \frac{R_{ij}}{\sqrt{V L N^{-1} \mu}} \right)}
\]

(8)

where \( Q(x) \) is the Gaussian \( Q \)-function defined by

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2} \xi^2} d\xi, \quad x \geq 0,
\]

(9)

and \( R = S^T S \) with \( S \) is an \( N \times L \) matrix of columns \( a \).

Average probability of error can be obtained by averaging the conditional probability of error (15) over the PDF, (10), of generalized K distribution as [7]

\[
P_e = F \cdot \frac{\int_{0}^{\frac{m+\mu}{2}} \frac{\Gamma(m+\mu)}{\Gamma(m)} \left( \frac{R_{ij}^2}{\Omega \Omega_o} \right)^{m+\mu} \frac{x^{m+\mu-1}}{x^{\frac{m+\mu}{2}} e^{-x^2/2}} dx}{\Gamma(m) \Gamma(\mu) \left( \frac{\Omega^2}{\Omega_o^2} \right)^{m+\mu}}
\]

(10)

where \( F = \frac{2}{\Gamma(m) \Gamma(\mu) \left( \frac{\Omega^2}{\Omega_o^2} \right)^{m+\mu}} \). Using the well known upper-bound approximation, called Chernoff bound, to \( Q(x) \), given by

\[
Q(x) \leq \frac{1}{2} e^{-x^2/2}
\]

(11)

the integral, \( I_1 \), of (15) can be expressed as

\[
I_1 = \int_{0}^{\frac{m+\mu}{2}} \frac{x^{m+\mu-1}}{x^{\frac{m+\mu}{2}} e^{-x^2/2}} \frac{\Gamma(m+\mu)}{\Gamma(m)} \left( \frac{R_{ij}^2}{\Omega \Omega_o} \right)^{m+\mu} \frac{x^{m+\mu-1}}{x^{\frac{m+\mu}{2}} e^{-x^2/2}} dx
\]

(12)

Now, by using [Eq. 6.631.3, 9] in solving (17), the average probability of error can be written as
\[
\bar{P}_e = F \cdot \frac{1}{2} \alpha - 0.5 \beta^2 \Gamma \left( \frac{1+d+l}{2} \right) \Gamma \left( \frac{1-d+l}{2} \right) \exp \left( \frac{\beta^2}{8d} \right) W_{0.5/d} \left( \frac{\beta^2}{4d} \right)
\]

where \(d = m - \mu, \ l = \frac{m+\mu}{2} - 1, \ \alpha = \frac{1}{\sqrt{\nu \sqrt{R^{-1} l_1}}}, \) and

\[
\beta = 2 \left( \frac{m \mu}{\Omega_o} \right) \text{ and } W_{\lambda, \gamma} (\cdot) \text{ is the Whittaker function [9].}
\]

IV. SIMULATION RESULTS

In this section, the performance of \(M\)-decorrelator is presented by performing Monte Carlo simulations by computing (18) for different values of Nakagami fading parameter and different shadowing levels. It is assumed that \(\Omega \approx 10 \text{ dB}\) in the simulations. Performance of decorrelating detector with different influence functions is shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5. In Fig. 2 and Fig. 3 (\(\varepsilon = 0.01 \& \kappa = 100\)), the average probability of error versus the signal-to-noise ratio (SNR) corresponding to the user 1 under perfect power control of a synchronous DS-CDMA system with six users \((L = 6)\) and a processing gain, \(N = 31\) is plotted for \(m = \mu = 1\) and \(m = \mu = 2\) respectively. Similarly, in Fig. 4 and Fig. 5 (\(\varepsilon = 0.1 \& \kappa = 100\)), average probability of error is plotted for \(m = \mu = 1\) and \(m = \mu = 2\) respectively. Simulation results show that the proposed \(M\)-estimator based detector performs well in the heavy-tailed impulsive noise compared to linear multiuser detector, minimax detector with Huber and Hampel estimators.

V. CONCLUDING REMARKS

Multiuser detection in DS-CDMA systems over fading and shadowing channels under impulsive noise environment is presented. An approximate closed-form expression for average probability of error of the decorrelating detector to detect BPSK (13) signals is derived. An \(M\)-estimator based multiuser detector is proposed and its probability of error is computed using the expression derived. Simulation results show that the proposed multiuser detector offers significant performance gain over the linear multiuser detector and the minimax decorrelating detector with Huber and Hampel \(M\)-estimators, in heavy-tailed impulsive noise.
Fig. 3. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed $M$-estimator in synchronous CDMA channel with impulse noise, $N = 31$, $\epsilon = 0.01$, $m = 2$, $\mu = 2$.

Fig. 4. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed $M$-estimator in asynchronous CDMA channel with impulse noise, $N = 31$, $\epsilon = 0.1$, $m = 2$, $\mu = 2$.

Fig. 5. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed $M$-estimator in asynchronous CDMA channel with impulse noise, $N = 31$, $\epsilon = 0.1$, $m = 1$, $\mu = 1$.

REFERENCES


