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Vinay Kumar Pamula

Corresponding Author:
icetet2014@yahoo.com

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JNTU Kakinada, Andhra Pradesh, India – 533 003

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Vinay Kumar Pamula
JNTU Kakinada, Andhra Pradesh, India – 533 003
pamulavk@yahoo.com

ABSTRACT

This paper presents a technique for blind multiuser detection in asynchronous direct sequence-code division multiple access (DS-CDMA) systems over Nakagami-*m* fading channels with impulsive noise. A new *M*-estimator is proposed and analyzed for robustifying the detector. A closed-from expression for average error rate of DPSK signals is derived. Average probability of error is computed to evaluate the performance of the new *M*-estimator based blind multiuser detector in comparison with the linear decorrelating detector, Huber and Hampel estimator based detectors. Simulation results show that the new *M*-estimator based detector performs well.

Keywords: CDMA; impulsive noise; M-estimator; multiuser detection; Nakagami-m distribution; probability of error.

I. INTRODUCTION

Blind multiuser detection can be used to reduce the inter-symbol interference (ISI) and also to improve system performance [1]. Subspace-based blind multiuser detection technique is known for its better performance among all the other blind methods. In [2], an improved subspace-based robust blind multiuser detection technique for synchronous and asynchronous CDMA systems over non-Gaussian channels is presented. A new adaptive algorithm for blind multiuser detection of coherent BPSK signals in DS-CDMA systems operating under non-Gaussian impulsive noise is presented in [1]. The problem of robust multiuser detection in non-Gaussian channels has been addressed in the literature [3], which was developed based on the Huber and the Hampel *M*-estimators, respectively. Differential non-coherent data detection for CDMA over flat-fading non-Gaussian channels with impulsive noise is presented in [2], [4]. Robust multiuser detection in synchronous DS-CDMA system with MRC receive diversity over Nakagami-*m* fading channel is presented in [5] by assuming that the modulation is binay PSK (BPSK). But, the differential modulation must be used to overcome the phase ambiguity [1], [4]. Recently, [6] considered the robust blind multiuser detection for synchronous DS-CDMA systems over Nakagami-*m* fading channels.

This paper extends the work of [6] to asynchronous DS-CDMA systems over Nakagami-*m* channels in the presence of heavy-tailed impulsive noise by employing subpace-based blind multiuser detection technique. An approximate expression for average probability of error of an *M*-decorrelator derived in [6] by using an approximation to Marcum-Q function. In this paper, a closed-form exact expression for average error rate is derived using [Eq. 3, 7]. This expression is used to compute the probability of error of linear decorrelating detector, the Huber and the Hampel estimator based detectors, and the newly (modified-Hampel) proposed *M*-estimator based detector. Simulation results shows that the new *M*-estimator based detector outperforms the linear decorrelating detector, Huber and Hampel estimator based detectors.

The remaining part of the paper is organized as: DS-CDMA system over multipath fading channel in impulsive noise environment is presented in Section II. In Section III, new *M*-estimator proposed to robustify the detector is presented. Section IV presents the subspace-based robust blind multiuser detection technique. An expression for average probability of error of *M*-decorrelator is derived in Section V. Section VI, presents the simulation results and finally, conclusions are drawn in Section VII.

II. SYSTEM MODEL

An asynchronous DS-CDMA system shared by *L*-users, where each user transmits information by modulating a signature sequence over a single-path Nakagami-*m* fading channel, is considered. The received signal over one symbol duration can be modeled, when the users are symbol and chip asynchronous, as [4]:

$$r(t) = \Re\left\{\sum_{l=1}^{L}\sum_{i=0}^{M-1} b_l(i)\alpha_l(t)e^{j\theta_l(t)} s_l(t-iT_s - \tau_l)\right\} + n(t) \quad (1)$$

Where $\Re\{\cdot\}$

denotes the real part, M is the number of data symbols per user in the data frame of interest, T_s is the symbol interval, $\alpha_l(t)$ is the time-varying fading gain of the l^{th} user's channel, $\theta_l(t)$ is the time-varying phase of the l^{th} user's channel, $b_l(i)$ is the i^{th} bit of the l^{th} user, $s_l(t)$ is the normalized signaling waveform of the l^{th} user, τ_l is the propagation delay and $n(t)$ is assumed as a zero-mean complex non-Gaussian noise given by [3]

$$f = (1-\varepsilon)\aleph(0,\upsilon^2) + \varepsilon\aleph(0,\kappa\upsilon^2) \quad (2)$$

With $\upsilon > 0, 0 \le \varepsilon \le 1$ and $\kappa \ge 1$.

Here $\aleph(0,\upsilon^2)$ represents the nominal background noise and the $\aleph(0,\kappa\upsilon^2)$ represents an impulsive component, with ε representing the probability that impulses occur. At the receiver, the received signal $r(t)$ is first filtered by a chip-matched filter and then sampled at the chip rate, $1/T_c$. The resulting discrete-time signal sample corresponding to the n^{th} chip of the i^{th} symbol is given by [4]

$$r_n(i) = \sqrt{\frac{2}{T_c}} \int_{iT_s+nT_c}^{iT_s+(n+1)T_c} r(t)e^{-j\omega_c t}dt, n = 1 \ldots N . \quad (3)$$

Assuming that the fading process for each user varies at a slower rate that the phase and amplitude can taken to be constant over the duration of a bit, (3) simplifies to [4]

$$r_j(i) = \frac{1}{\sqrt{N}}\sum_{l=1}^{L}\left[g_l(i)b_l(i)h_j^l(0) + g_l(i-1)b_l(i-1)h_j^l(1)\right] + w_j(i)$$

(4)

Where

$$h_j^l(0) \triangleq \begin{cases} 0, & j < m_l \\ (1-\pi_l)a_{j-m_l}^l, & j = m_l \\ \pi_l a_{j-m_l-1}^l + (1-\pi_l)a_{j-m_l}^l, & j > m_l \end{cases} \quad (5)$$

$$h_j^l(1) \triangleq \begin{cases} \pi_l a_{j+N-m_l-1}^l + (1-\pi_l)a_{j+N-m_l}^l, & j < m_l \\ \pi_l a_{j-m_l+N-1}^l, & j = m_l \\ 0, & j > m_l \end{cases} \quad (6)$$

With $m_l \equiv \left\lfloor \frac{\tau_l}{T_c} \right\rfloor$, $\pi_l \equiv \frac{\tau_l}{T_c} - m_l$

and

$$w_j(i) = \sqrt{\frac{2}{T_c}} \int_{iT_s+jT_c}^{iT_s+(j+1)T_c} n(t)e^{-j\omega_c t}dt . \quad (7)$$

Let $\underline{\boldsymbol{r}}(i) \triangleq \left[\boldsymbol{r}_0(i),\ldots,\boldsymbol{r}_{N-1}(i)\right]^T$, $\underline{\boldsymbol{w}}(i) \triangleq \left[\boldsymbol{w}_0(i),\ldots,\boldsymbol{w}_{N-1}(i)\right]^T$

and

$$\underline{\boldsymbol{\theta}}(i) \triangleq \frac{1}{\sqrt{N}}\left[\boldsymbol{b}_1(i)\boldsymbol{g}_1(i),\ldots,\boldsymbol{b}_L(i)\boldsymbol{g}_L(i)\right]^T .$$

by stacking k successive data samples, the following quantities can be defined [2], [4],

$$\underline{r}_k(i) \cong \begin{bmatrix} \underline{r}(i) \\ \cdot \\ \cdot \\ \cdot \\ \underline{r}(i+k-1) \end{bmatrix}_{kN}, \quad \underline{\theta}_k(i) \cong \begin{bmatrix} \underline{\theta}(i-1) \\ \underline{\theta}(i) \\ \cdot \\ \cdot \\ \underline{\theta}(i+k-1) \end{bmatrix}_{(k+1)N} \quad (8)$$

$$\underline{w}_k(i) \cong \begin{bmatrix} \underline{w}(i) \\ \cdot \\ \cdot \\ \cdot \\ \underline{w}(i+k-1) \end{bmatrix}_{kN}$$

And $\underline{r}_k(i)$ can be written as

$$\underline{r}_k(i) = \underline{\boldsymbol{H}}_k(i)\underline{\theta}_k(i) + \underline{w}_k(i), \quad (9)$$

Where

$$\underline{\boldsymbol{H}}_k(i) \cong \begin{bmatrix} \underline{\boldsymbol{H}}(1) & \underline{\boldsymbol{H}}(0) & \underline{0} & \cdot & \cdot & \underline{0} \\ \underline{0} & \cdot & & & \cdot & \underline{0} \\ \cdot & & \cdot & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ \underline{0} & \cdot & \cdot & \cdot & \underline{\boldsymbol{H}}(1) & \underline{\boldsymbol{H}}(0) \end{bmatrix} \quad (10)$$

With

$$\underline{\boldsymbol{H}}(0) \cong \begin{bmatrix} h_1^l(0) & \cdot & \cdot & \cdot & h_1^L(0) \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ h_N^l(0) & \cdot & \cdot & \cdot & h_N^L(0) \end{bmatrix} \quad (11)$$

And

$$\underline{\boldsymbol{H}}(1) \cong \begin{bmatrix} h_1^l(1) & \cdot & \cdot & \cdot & h_1^L(1) \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ h_N^l(1) & \cdot & \cdot & \cdot & h_N^L(1) \end{bmatrix}. \quad (12)$$

The parameter k is called the smoothing factor and is chosen such that the matrix $\underline{\boldsymbol{H}}_k$ has full column rank.

III. M- ESTIMATOR

There exist in the literature a number of different approaches to the robust estimation problem, and the *M*-estimation is one of the most sophisticated approaches to this problem [8]. An important class of robust estimators is *M*-estimators and they can resist outliers without preprocessing the data [9]. In *M*-estimates, unknown parameters $\underline{\theta}_1, \underline{\theta}_2, \ldots \underline{\theta}_L$ are solved by minimizing a sum of function $\rho(\cdot)$ of the residuals [4]

$$\underline{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta} \in C^L} \sum_{n=1}^{N} \left\{ \rho\left[\Re\left(\boldsymbol{r}_n(i) - \sum_{l=1}^{L} [\underline{A}]_{nl}\, \theta_l(i) \right) \right] + \rho\left[\Im\left(\boldsymbol{r}_n(i) - \sum_{l=1}^{L} [\underline{A}]_{nl}\, \theta_l(i) \right) \right] \right\} \quad (13)$$

where ρ is a symmetric positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square, $r_n(i)$ and $\theta_l(i)$ are the n^{th} and l^{th} elements of the vectors $\underline{r}(i)$ and $\underline{\theta}(i)$ respectively, $[\underline{A}]_{nl}$ is the nl^{th} element of the matrix $\underline{A}$ [see 4 for details], and $\Im(\cdot)$ denotes imaginary part. Since least squares (LS) estimate is very sensitive to the tail behavior of the noise density, linear decorrelating multiuser detector is also sensitive to the tail behavior of the noise distribution. Hence, a new *M*-estimator [10] is used to deal with heavy-tailed noise. The influence function of new *M*-estimator is given by [10], [11]

$$\psi_{PRO}(\zeta) = \begin{cases} \zeta & for\ |\zeta| \le a \\ a\,\mathrm{sgn}(\zeta) & for\ a < |\zeta| \le b \\ \dfrac{a}{b}\zeta \exp\left(1 - \dfrac{|\zeta|^2}{b^2}\right) & for\ |\zeta| > b \end{cases}. \quad (14)$$

The choice of the constants a $(=\kappa\upsilon^2)$ and b $(= 2\kappa\upsilon^2)$ depends on the robustness measures derived from the influence function.

IV. ROBUST BLIND MULTIUSER DETECTION

In robust blind multiuser detection for asynchronous channels the signal subspace components of the covariance matrix of the signal $\underline{r}_k(i)$ are computed as [2], [4]

$$\underline{\boldsymbol{C}} \cong E\left[\underline{\boldsymbol{r}}_l(i)\underline{\boldsymbol{r}}_l(i)^T\right] = \underline{\boldsymbol{H}}_k(i)\underline{W}_k^2\underline{\boldsymbol{H}}_k^T + \upsilon \underline{\boldsymbol{I}}_N = \underline{U}_s \underline{\Lambda}_s \underline{U}_s^{\,T} + \upsilon^2 \underline{U}_n \underline{U}_n^{\,T}. \quad (15)$$

Here, the signal subspace has dimension of L $(m+1)$. Then, the robust estimation in the signal subspace follows as

$$r^k(i) \cong \psi\left(r - U_s \xi^k(i)\right) \quad (16)$$

$$\xi^{k+1}(i) = \xi^{k}(i) + \frac{1}{\mu} U_s^T r^k(i) \qquad (17)$$

Where μ is a step-size parameter chosen as $\frac{1}{\mu} = \nu^2$.

The estimate of the product of the amplitude and the data bit $\theta_l(i) = W_l b_l(i)$ of the l^{th} user is given by [4]

$$\theta_l(i) = \sum_{j=1}^{L(k+1)} \frac{u_j^T h_l}{\lambda_j} \xi_l(i), l=1,2,...,L \qquad (18)$$

Where $h_l \cong \left[h_l^T \quad 0_{N(k-2)}^T \right]^T$ and finally, the l^{th} user data bit is demodulated as

$$\hat{b}_l(i+1) = \operatorname{sgn}\left\{ \Re\left[\theta_{k,2L+l}(i) \theta^*_{k,2L+l}(i) \right] \right\}. \qquad (19)$$

V. AVERAGE PROBABILITY OF ERROR OF M-DECORRELATOR

The asymptotic probability of error of non-coherent DPSK demodulator can be evaluated by averaging the conditional probability [4]

$$P\left(\Re\left\{ \hat{\theta}_l(i) \hat{\theta}_l^*(i-1) \right\} < 0 \middle| \Delta\phi_l(i) = 0, x, y \right)$$
$$= Q_M\left(\sqrt{\frac{1}{2\sigma^2_{\hat{\theta}_l}}} x, \sqrt{\frac{1}{2\sigma^2_{\hat{\theta}_l}}} y \right) - \frac{1}{2} I_o\left(\frac{xy}{2\sigma^2_{\hat{\theta}_l}} \right) e^{-\frac{x^2+y^2}{4\sigma^2_{\hat{\theta}_l}}} \qquad (20)$$

Over the joint probability density function (PDF) of the random variables x and y [4].

Here

$$\sigma^2_{\hat{\theta}_l} \triangleq \frac{1}{N} \nu^2 \left[\underline{R}^{*-1} \right]_{ll} \qquad (21)$$

With

$$\nu^2 = \frac{\int \psi^2(u) f(u) du}{\left[\int \psi'(u) f(u) du \right]^2} \qquad (22)$$

And $\underline{R}^*$ is the cross-correlation matrix of the random infinite-length signature waveforms of the L users, $Q_M(\cdot,\cdot)$ is the Marcum's Q-function and $I_o(\cdot)$ is the modified Bessel function of the first kind with order zero. Here, it is assumed that the random variables x and y are statistically independent and Nakagami-m distributed with PDF [12]. By following the steps as in [6] and using [Eq. 3, 7], the average probability of error of DPSK signals can be derived as (23) and shown at the end of the paper.

In (23), $H = \frac{(1/4\sigma^2_{\hat{\theta}_l})/(4m/\Omega + 1/\sigma^2_{\hat{\theta}_l})}{(m/\Omega + 1/4\sigma^2_{\hat{\theta}_l})}$, ${}_2F_1(\cdot)$ is the Gauss hyper- geometric function and $\Gamma(\cdot)$ is the gamma function [13],

$$\Omega = \frac{1}{N}\left\{ \nu^2 \left[\underline{R}^{*-1} \right]_{ll} + 2SNR_l \sigma^2 (1-\rho_l) \right\}, \qquad (23)$$

$\sigma^2 = (1-\varepsilon)\upsilon^2 + \varepsilon\kappa\upsilon^2$ (With υ^2 and $\kappa\upsilon^2$, respectively, as the variance of in-phase and quadrature components of noise samples) and

$$SNR_l \triangleq \frac{E\left[\left| \boldsymbol{g}_l(i) \right|^2 \right]}{\sigma^2}. \qquad (24)$$

VI. SIMULATION RESULTS

In this section, the performance of M-decorrelator is presented by computing (23) for different values of Nakagami fading parameter and different order of diversity. It is assumed that the channel model is a lightly damped second-order autoregressive (AR) process given by [4] with the bit rate, the pole radius, and the spectral peak frequencies are T_b = 10 kbps, r_d = 0.998 and f_p = 80 Hz, respectively. Further, it is also assumed that M = 1 in evaluating average probability of error, (23).

Performance of decorrelating detector with different influence functions is shown in Fig. 1 and Fig. 2. In Fig. 1, the average probability of error versus the signal-to-noise ratio (SNR) corresponding to the user 1 under perfect power control of an asynchronous DS-CDMA system with six users (L = 6) and a large processing gain, N = 127 is plotted for m = 1 and D = 1. Similarly, in Fig. 2, average probability of error is plotted for m = 2 and D = 2. The simulation results reveal that the increase in diversity order (from 1 to 2) improves the detector performance. Simulation results also reveals that the proposed M-estimator outperforms the linear decorrelating detector and minimax decorrelating detector (both with Huber and Hampel estimators), even in highly impulsive noise. Moreover, this performance gain increases as the SNR increases.

VII. CONCLUDING REMARKS

Blind multiuser detection in asynchronous DS-CDMA systems over Nakagami-*m* fading channels in an impulsive noise environment is presented. A new *M*-estimator (modified-Hampel) based blind multiuser detection technique is presented and its performance is analyzed by deriving a closed-form expression for average error rate. This expression is used to compute the average probability of error of the *M*-decorrelator with least-squares, Huber, Hampel and proposed *M*-estimators. Simulation results show that the blind multiuser detector offers significant performance gain over the linear multiuser detector and the minimax decorrelating detectors with Huber and Hampel *M*-estimators, in impulsive noise. Effect of fading parameter and diversity order on the performance of decorrelator is also studied.

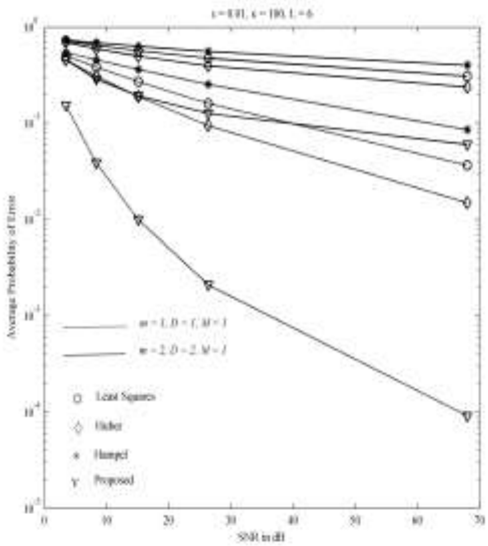

Fig. 1. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed *M*-estimator in asynchronous DS-CDMA channel with moderate (ε = 0.01) impulsive noise.

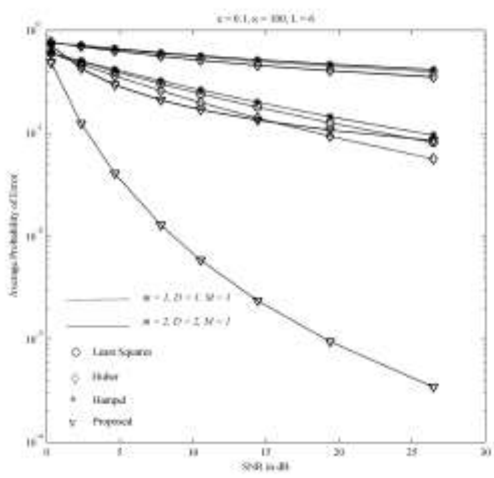

Fig. 2. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed *M*-estimator in asynchronous DS-CDMA channel with highly (ε = 0.1) impulsive noise.

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$$\bar{P}_e^l = \frac{(m/\Omega)^{mD}\Gamma(mD+M)}{\left(m/\Omega+1/4\sigma^2_{\hat{\theta}_l}\right)^{mD+M}} \left[\begin{array}{l} \left(1/4\sigma^2_{\hat{\theta}_l}\right)^M {}_2F_1\left(1,mD+M;mD+1;(m/\Omega)/(m/\Omega+1/4\sigma^2_{\hat{\theta}_l})\right)/(mD)! \\ +\left(1/2\sigma^2_{\hat{\theta}_l}\right)\sum_{i=0}^{mD-1} \frac{1}{M!(m/\Omega)^{mD-i}2^{M-i+1}\left(2m/\Omega+1/2\sigma^2_{\hat{\theta}_l}\right)^{i+1}} {}_2F_1(i+1,mD+M;H) \end{array} \right] \tag{25}$$

$$+ 0.5\, {}_2F_1\left(mD, mD; 1; -1/(16\sigma^4_{\hat{\theta}_l})(\Omega/m)^2\right)$$